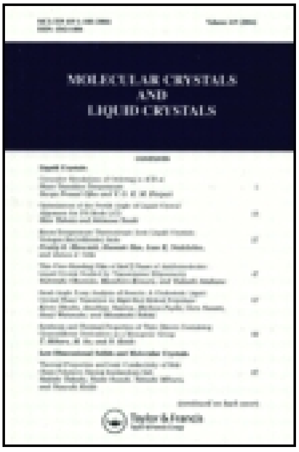

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Chiral-Nematic Layers With Asymmetric Boundary Coupling

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The influence is described of asymmetric boundary conditions on the field-induced director patterns of twisted chiral-nematic layers. In the weak anchoring case the surface tilt angles as well as the coupling constants may be different. The different boundary conditions break the symmetry of the director patterns. The position of the extreme director tilt becomes voltage dependent.

Keywords: electrooptic, supertwist, asymmetric boundary

INTRODUCTION

A lot of experimental and theoretical work has been published in the field of supertwisted chiral-nematic layers. The higher twist angle improves the electro-optical properties of such layers and high information content applications become feasible. Depending on the applied electro-optical effect different orientation technologies are required.[1] In all commercially available LCDs almost rigid boundary conditions are realized. That means the first molecular layer of the liquid crystal takes the tilt angle which is determined by the surface treatment.

From a theoretical point of view, we know that weak anchoring can improve the electro-optical properties. Generally the boundary conditions are assumed to be the same at both side of the liquid crystal.

In the present paper we investigate the consequences of asymmetrical boundary conditions. Asymmetry means, that the tilt angles at the boundaries are different, for weak anchoring even the coupling constants may be different.

SYMMETRY

In the case of equal boundary conditions at both surfaces the director patterns are symmetrical with respect to the mid-plane. Therefore calculations are only performed for one half space.

In the present paper the asymmetrical boundary conditions produce asymmetrical

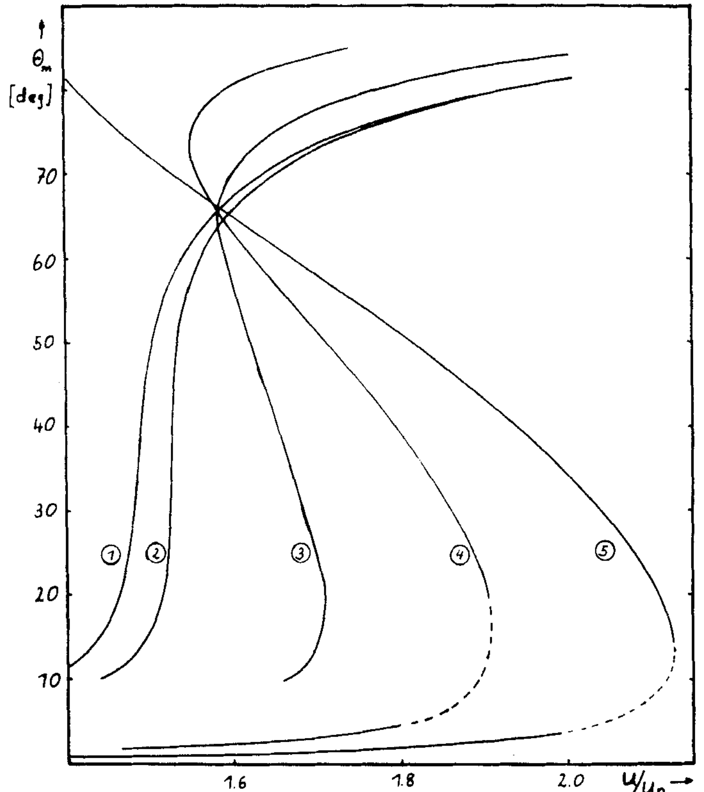

FIGURE 1 θ_m versus reduced voltage for chiral-nematic layers with different total twist angles ϕ_0; (1)-180°, (2)-180°, (3)-210°, (4)-240°, (5)-270°.

director patterns. The extreme director tilt no longer exists in the mid-plane. Moreover the location of the maximum (minimum) depends on the boundary conditions as well as on the applied voltage. Because of the asymmetry a critical voltage for which the tilt angle $\theta(z)$ has a constant value throughout the sample does not exist. Instead of that, we find a voltage region where the director tilt varies monotonically from one boundary to the other. Within that region a voltage exists for which the director tilt varies linearly with position.

THEORY

Several authors have already described the electric field induced director patterns in high twist configurations using the continuum theory.[2,3]

We consider a chiral-doped nematic layer of thickness d between two electrodes. The chiral additive is added in such a quantity that the intrinsic pitch p_0 corresponds

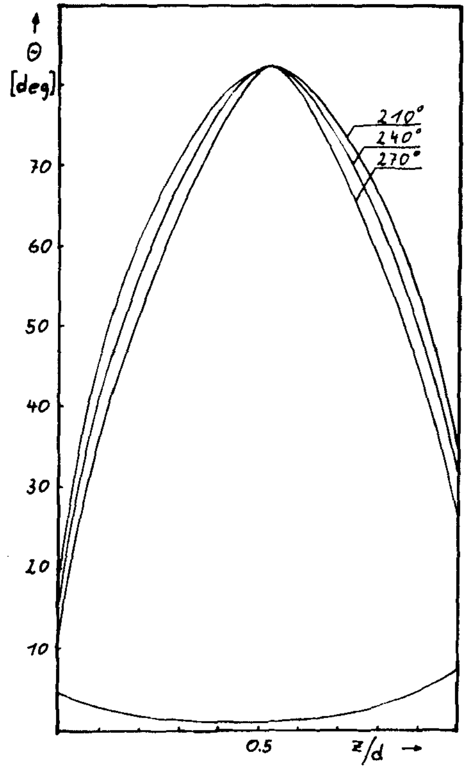

FIGURE 2a Tilt profiles for three curves of Figure 1.

to the total twist ϕ_0 induced by the boundary conditions. The following material parameters come into play

$$\alpha = (k_{33} - k_{22})/k_{22} \qquad \chi = (k_{33} - k_{11})/k_{11}$$

$$\gamma = (e_2 - \varepsilon_1)/\varepsilon_1$$

k_{ii}—elastic constants, ε_i—dielectric constants.

In the weak anchoring case the equilibrium value of the surface tilt of the liquid crystal molecules θ_i is different from the preferential direction θ_{pi} and depends on the boundary conditions and on the applied voltage u.

The elastic coupling of the molecules at the substrate surfaces is of the Rapini-Papoular type[4] (azimuthal coupling). With respect to the polar angle, the coupling is assumed to be rigid. Therefore the total twist angle ϕ_0 remains constant.

We find the equilibrium states of the chiral-nematic layer by solving the variational problem for the free energy. The Euler-Lagrange equations look similar to those in the strong anchoring case. In addition we get two equations describing the different boundary conditions. The first integrals of the differential equations

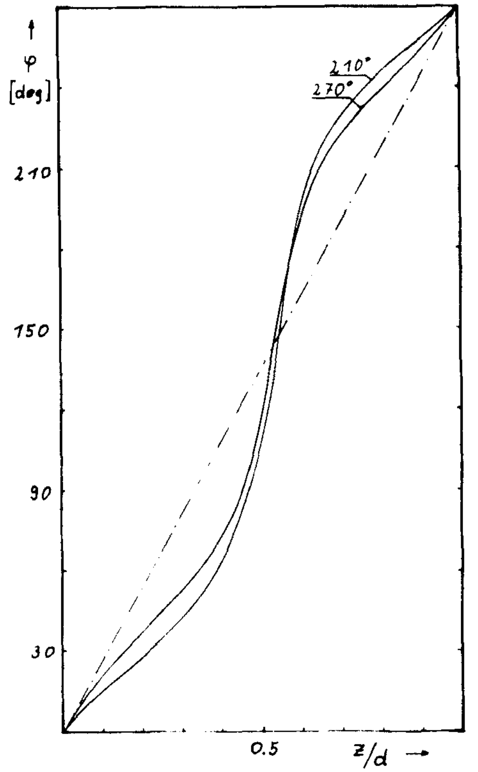

FIGURE 2b Twist profiles for two curves of Figure 2.

can easily be written down. To determine the constants of integration the existence of an extremum for the director tilt θ_m is assumed. The derivative of the director twist at the same position is denoted by β.

In the asymmetrical case these quantities have a generalized meaning. If the director angle increases (decreases) monotonically throughout the sample, we find the extremum to be located outside the sample. Then θ_m represents rather a mathematical auxiliary quantity which describes the behaviour of the director. After all we can establish three coupled equations to determine the unknown quantities η, β and $\lambda = \beta/D_z\sqrt{k_{22}*\varepsilon_1}$,

where $\eta = \sin^2\theta_m$ and D_z is the dielectric displacement.

$$\sqrt{\eta} = \phi_0 * \left[\int \frac{g_m + t_0 * \beta^{-1}(e_m - e)}{w * g * \sqrt{1 - \eta x^2}}\, dx\right]^{-1} \tag{1}$$

$$\beta = \frac{\sqrt{\eta}}{d}\int \frac{dx}{w\sqrt{1 - \eta x^2}} \tag{2}$$

$$\lambda = \frac{u_p * \sqrt{\gamma * \eta}}{\pi * u}\sqrt{k_{22}/k_{11}}\int \frac{h * dx}{w\sqrt{1 - \eta x^2}} \tag{3}$$

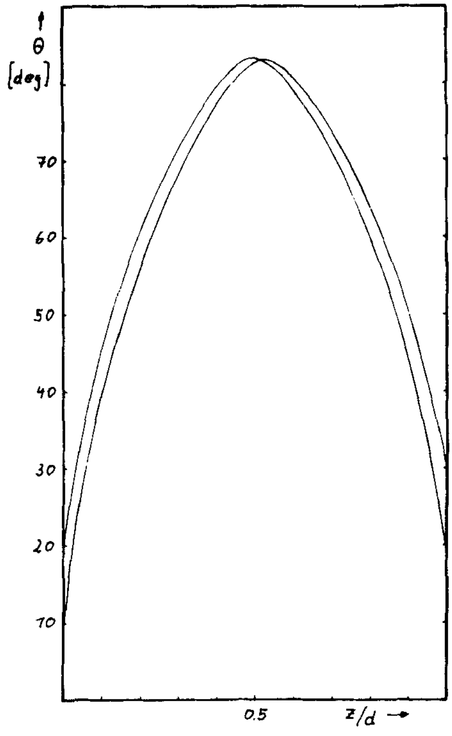

FIGURE 3a Tilt profiles of a symmetric layer ($\theta_{p1} = \theta_{p2} = 20°$) and of an asymmetric layer ($\theta_{p1} = 10°$, $\theta_{p2} = 30°$) with strong surface coupling.

with the following abbreviations

$$w = \sqrt{k_{22}/k_{11}}/\sqrt{f*g}\ \{g*g_m + g/\lambda^2*(h - h_m) - [g_m + t_0/\beta*(e_m - e)]^2\}^{1/2}$$

$$t_0 = 2\pi/p_0$$

$$e = e(\theta) = \sin^2\theta - 1$$

$$f = f(\theta) = 1 + \chi* \sin^2\theta$$

$$g = g(\theta) = \cos^2\theta*(1 + \alpha* \sin^2\theta)$$

$$h = h(\theta) = (1 + \gamma \sin^2\theta)^{-1}$$

$$e_m = e(\theta)_m \text{ a.s.o.}$$

In the present model we have two additional equations to describe the different

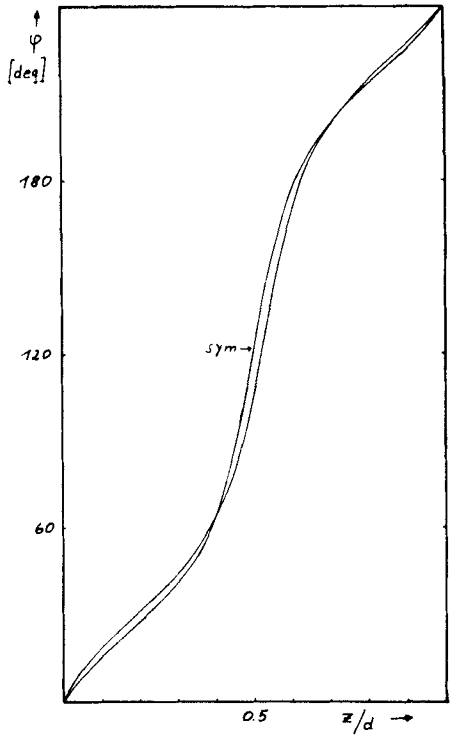

FIGURE 3b Twist profiles for the layers of Figure 3a.

situations at both boundary surfaces.

$$\sin 2(\theta_1 - \theta_{p1}) = -\frac{2}{\pi} - r_1 * f(\theta_1) * w(\theta_1) \tag{4}$$

$$\sin 2(\theta_2 - \theta_{p2}) = -\frac{2}{\pi} - r_2 * f(\theta_2) * w(\theta_2) \tag{5}$$

Both the preferential tilt angle θ_{pi} and the establishing surface tilt θ_i occur in the last two equations. Therefore it is possible to reduce the weak anchoring problem to the rigid coupling case without solving simultaneously the above five equations. Starting point is an initial guess for the surface tilt θ_i. Then Equations (1)–(3) are solved like in the rigid coupling case. After that we determine the corresponding preferential tilt angle from Equations (4) and (5). If it is not correct the initial value must be varied a.s.o.

In the present paper we choose the following material parameters if nothing

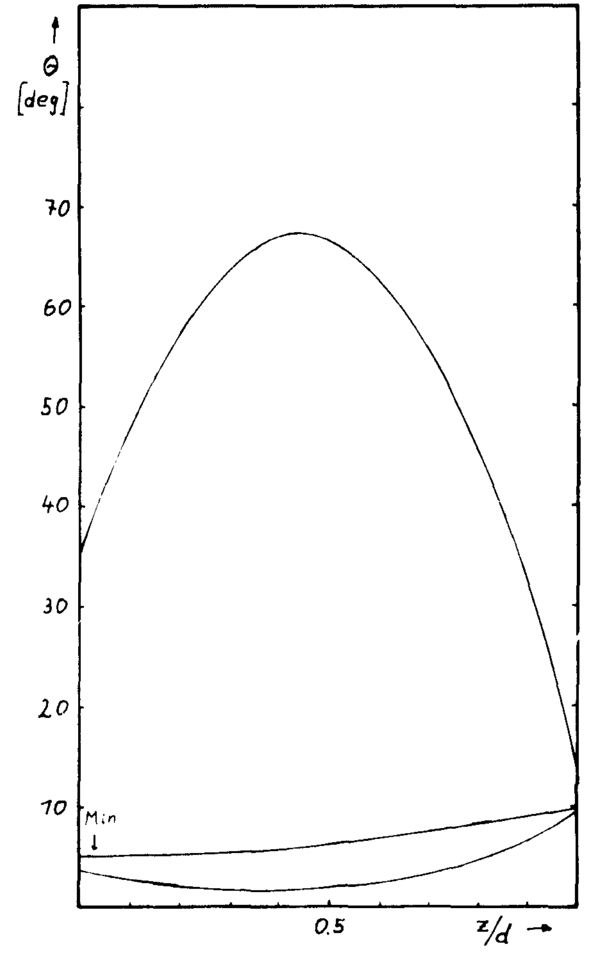

FIGURE 4 Tilt profiles of a 240° twisted layer with highly asymmetrical anchoring conditions (r_1 = 0.4, r_2 = 0.05, θ_{p1} = 5°, θ_{p2} = 10°).

otherwise is remarked

$$\alpha = 3.0 \qquad \chi = 1.0 \qquad \gamma = 2.0$$

RESULTS AND DISCUSSION

Figure 1 shows the voltage dependence of the maximum director tilt for layers having different total twist angles ((2)-180°, (3)-210°, (4)-240°, (5)-270°). Curve (1) has different elastic constants α = 2.6 and χ = 0.8 but elsewhere the same parameters like curve (2). The boundary conditions are for all curves of Figure 1 the same

$$r_1 = 0.1 \qquad \theta_{p1} = 5.0°$$

$$r_2 = 0.2 \qquad \theta_{p2} = 10.0°$$

Voltages are expressed on a reduced scale, normalized by the threshold voltage of a planar Fréedericksz cell u_p.

In Figure 2a we have displayed the tilt angle profiles for three curves of Figure 1. The corresponding twist profiles are to be seen in Figure 2b.

In Figure 3a the tilt angle profiles for a symmetrical layer with $\theta_{p1} = \theta_{p2} = 20.0°$ and for an asymmetrical layer with $\theta_{p1} = 10.0°$ and $\theta_{p2} = 30.0°$ in the strong anchoring case are displayed. Both curves become not congruent if one of them is shifted in the z-direction. The symmetrical curve has a slightly higher maximum at the same reduced voltage ($v_{re} = 1.8$). The twist angle profile is to be seen in Figure 3b. In Figure 4 we have displayed the director tilt of an asymmetrical layer at different reduced voltages with

$$r_1 = 0.4 \quad r_2 = 0.05 \quad \theta_{p1} = 5.0° \quad \theta_{p2} = 10.0° \quad \text{and} \quad \phi_0 = 240°.$$

The quite different boundary conditions produce highly asymmetrical director profiles.

Therefore it happens, that for low voltages the surface tilt at the right boundary is greater, than at the left boundary and for higher voltages the situation is vice versa. For the upper and the lower curve in the picture the reduced voltage is $v_{re} = 1.434$. The lower curve is the stable one. For $v_{re} = 1.821$ we have a nearly linear profile. Looking carefully we find a shallow minimum near to the left side. For a slightly higher voltage the minimum coincides with the preferential tilt angle at the left boundary. Then the tilt angle varies monotonically throughout the whole sample. The same is truth for the macroscopical properties. If such a configuration can be realized experimentally new applications become possible.

Empirically we have found the following relation between the tilt angles and the coupling constants

$$\frac{\theta_{p2} - \theta_2}{\theta_{p1} - \theta_1} = \frac{r_2}{r_1}.$$

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